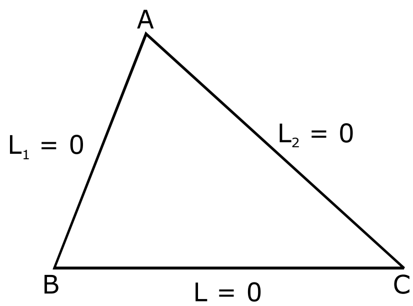
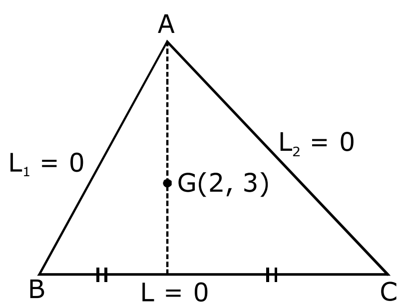
EXERCISE – IV HINTS & SOLUTIONS

Sol.1 Centroid $\rightarrow$ G
Orthocentre $\rightarrow$ H
Circumcentre $\rightarrow$ s



(a) $L_1 : 2x - y = 0$
$L_2 : x + y = 3$ & G(2, 3)



$\Rightarrow$ A (1, 2)

$\dfrac{AG}{GD} = \dfrac{2}{1}$ Let D (x_1, y_1)

$\Rightarrow \dfrac{2x_1+1}{2+1} = 2$ & $\dfrac{2y_1+2}{2+1} = 3$

$\Rightarrow x_1 = \dfrac{5}{2}$ & $y_1 = \dfrac{7}{2}$ D $\equiv \left(\dfrac{5}{2}, \dfrac{7}{2}\right)$

Let B (a, 2a) & C(b, 3 – b)
D is mid point of BC

$\dfrac{a+b}{2} = \dfrac{5}{2}$ & $\dfrac{2a+3-b}{2} = \dfrac{7}{2}$

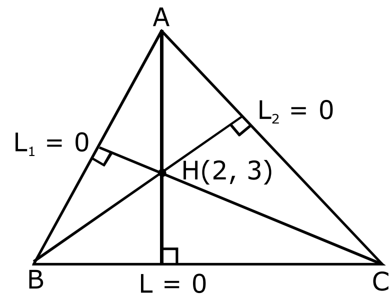
$$\begin{array}{rl} & a + b = 5 \\ \& & 2a - b = 4 \\ \hline & 3a = 9 \end{array}$$

$\Rightarrow$ a = 3 B $\equiv$ (3, 6)
b = 2 C $\equiv$ (2, 1)

Line BC is $y - 1 = \dfrac{6-1}{3-2}(x - 2)$

$5x - y = 9$ Slope is 5

(b) $L_1 : 2x + y = 0$
$L_2 : x - y + 2 = 0$

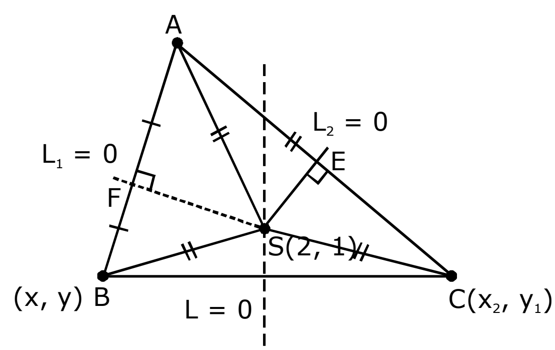


Altitudes from B & C
Passing through H(2, 3)
$L_C : x - 2y = -4$ & $L_B : x + y = 5$
Intersection of L_B & L_1
$x = -5$ & $y = 10$ B(–5, 10)
Intersection of L_C & L_2
y = 2 & x= 0 C(0, 2)
Line BC is

$(y - 2) = \dfrac{10-2}{-5}(x)$

y intercept is (If x = 0)
y = 2

(c) $L_1 : x + y - 1 = 0$
$L_2 : 2x - y + 4 = 0$



Intersection point A (–1, 2)
Line SE
$x + 2y - 4 = 0$
& Line SF $x - y - 1 = 0$
B, C is image of A.w.r. to., SF & SE respectively.

$\dfrac{x_1+1}{1} = \dfrac{y_1-2}{-1} = \dfrac{-2(-1-2-1)}{1^2-(-1)^2}$

$x_1 = -1 + 4 = 3$ & $y_1 = 2 - 4 = -2$
$\Rightarrow$ B (3, –2)

$$\frac{x_2+1}{1} = \frac{y_2-2}{2} = -\frac{-2(-1+4-4)}{1^2+2^2}$$

$$x_2 = -1 + \frac{2}{5} = \frac{-3}{5},\ y_2 = 2 + \frac{4}{5} = 14$$

$$\Rightarrow C\left(\frac{-3}{5}, \frac{14}{5}\right)$$

Line BC

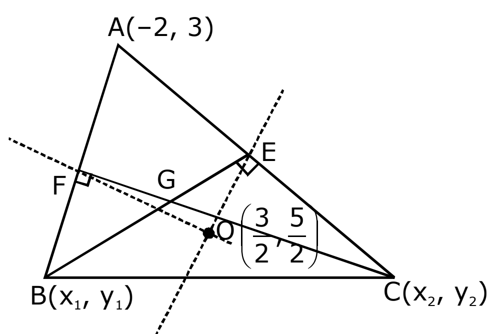
$$L : (y+2) = \frac{\frac{14}{5}+2}{\frac{-3}{5}-3}(x-3)$$

$$y + 2 = \frac{4}{-3}(x-3)$$

x intercept is (put y = 0)

$$x = -\frac{2\times 3}{4} + 3 \qquad \Rightarrow x = \frac{3}{2}$$

Sol.2 Perpendicular of side AB & AC are
$x - y - 4 = 0$
$2x - y - 5 = 0$



line OE should be

$$2x - y - C_1 = 0 \qquad \Rightarrow 2x - y - \frac{1}{2} = 0$$

$4x - 2y - 1 = 0$
Line of should be $\quad x - y + 1 = 0$
Image of A w.r.t. to OE & OF is C & B respectively

$$\frac{(x_1+2)}{1} = \frac{(y_1-3)}{-1} = \frac{-2(-2-3+1)}{1^2+1^2} = 4$$

$\Rightarrow x_1 = 2, y_1 = -1 \quad$ B (2, −1)

$$\& \frac{(x_2+2)}{4} = \frac{(y_2-3)}{-2} = \frac{-2(-8-6-1)}{16+4} = \frac{3}{2}$$

$\Rightarrow x_2 = 4 \qquad \& \ y_2 = 0 \qquad \Rightarrow$ C(4, 0)

Median BE is $y + 1 = -\frac{5}{2}(x-2)$

$\Rightarrow 5x + 2y = 8$

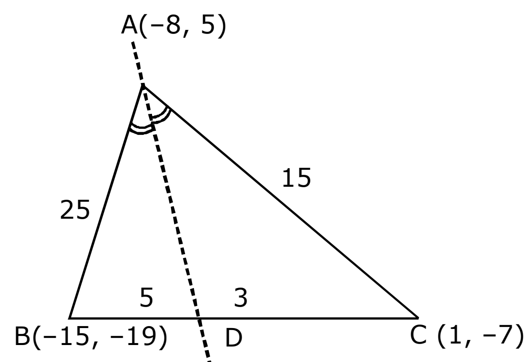
& CF is $(y-1) = -\frac{1}{4}x \quad \Rightarrow x + 4y = 4$

Sol.3 Angle bisector AC is $ax + 2y + c = 0$

$AB = \sqrt{7^2+24^2} = 25$

$AC = \sqrt{9^2+12^2} = 15$

$$\frac{AB}{AC} = \frac{BD}{CD} = \frac{25}{15} = \frac{5}{3}$$



D is divdes BC in the ratio 5 : 3

Coordinate of D $\left(\frac{-45+5}{8}, \frac{-57-35}{8}\right)$

$$\equiv D\left(-5, \frac{-23}{2}\right)$$

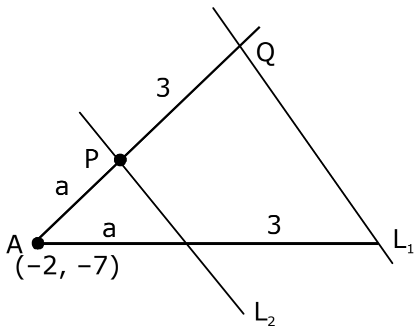
Line AD is $y - 5 = \dfrac{\frac{-23}{2}-5}{-5+8}(x+8)$

$$\Rightarrow y - 5 = \frac{-33}{2.3}(x+3)$$

$\Rightarrow 2y - 10 = -11x - 28$
$\Rightarrow 11x + 2y + 78 = 0$
$a = 11, c = 78$

Sol.4 $L_1 : 4x + 3y - 12 = 0$
$L_2 : 4x + 3y - 3 = 0$

$$\frac{x+2}{\cos\theta} = \frac{y+7}{\sin\theta} = a$$



$P(a\cos\theta - 2,\ a\sin\theta - 7)$ lies on L_2

$4a\cos\theta - 8 + 3a\sin\theta - 21 - 3 = 0$

$a(4\cos\theta + 3\sin\theta) = 32$ ....(i)

$\frac{x+2}{\cos\theta} = \frac{y+7}{\sin\theta} = (a+3)$

$Q((a+3)\cos\theta - 2, (a+3)\sin\theta - 7)$ lies on L_1

$4(a+3)\cos\theta - 8 + 3(a+3)\sin\theta - 21 = 0$

$32 + 3 \cdot \frac{32}{a} = 41 \Rightarrow a = \frac{32}{3}$

From (i)

$4\cos\theta + 3\sin\theta = 3$

$$\frac{4(1-\tan^2\frac{\theta}{2}) + 3.2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = 3$$

$\Rightarrow\ 7\tan^2\frac{\theta}{2} - 6\tan\frac{\theta}{2} - 1 = 0$

$\tan\frac{\theta}{2} = 1$ or $\tan\frac{\theta}{2} = -\frac{-1}{7}$

$$\Rightarrow \tan\theta = \frac{2.1}{1-1^2} \text{ or } \tan\theta = \frac{2\left(-\frac{1}{4}\right)}{1-\left(\frac{-1}{7}\right)^2} = \frac{-7}{24}$$

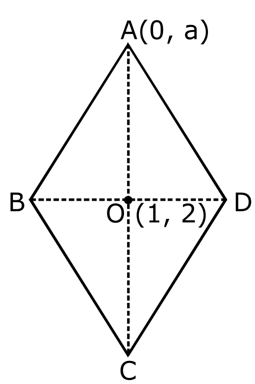
$\theta = 90^o \Rightarrow$ poarallel to y-axis

Lines $x = -2 \Rightarrow x + 2 = 0$

$\&\ y + 7 = -\frac{-7}{24}(x+2)$

$7x + 24y + 154 = 0$

Sol.5 Sides parallel to
$y = x + 2,\ y = 7x + 3$
A lie on y-axis
let (0, a)
Sides which passing through are
$y = x + a$
$\&\ y = 7x + a$
or $x - y + a = 0$
$\&\ 7x - y + a = 0$
diagonals should be (angle bisectors)



$$\frac{x-y+a}{\sqrt{2}} = \pm\left(\frac{7x-y+a}{\sqrt{50}}\right)$$

$5x - 5y + 5a = \pm(7x - y + a)$

passing through (1, 2)

$-5 + 5a = \pm(5 + a)$

$-5 + 5a = 5 + a$ or $-5 + 5a = -5 - 9$

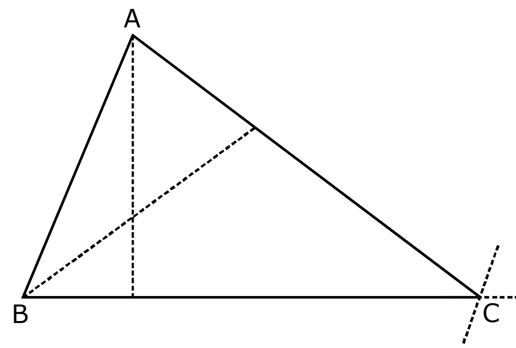
$5a = 10$ or $a = 0$

$a = \frac{5}{2}$

A should be (0, 0) or $\left(0, \frac{5}{2}\right)$

Sol.6 $AB_1 : x + y - 5 = 0 \Rightarrow m_2 = -1$

$BC_2 : x + 7y - 7 = 0 \Rightarrow m_1 = -\frac{1}{7}$



$AC_3 : 7x + y + 14 = 0 \Rightarrow m_3 = -7$

$-\frac{1}{7} > -1 > -7$

$m_1 = \frac{-1}{7},\ m_2 = -1,\ m_3 = -7$

$\tan B = \frac{\frac{-1}{7}+1}{1+\frac{1}{7}} = \frac{6}{8} = \frac{3}{4}$ $B \to$ acute $< \frac{\pi}{4}$

$\tan A = \frac{-1+7}{1+7} = \frac{6}{8} = \frac{3}{4}$ $A \to$ acute $< \frac{\pi}{4}$

$\tan C = \frac{-1+7}{1+7} < 0$

ΔABC is isosceles traingle

Make constant positive sign in lines

Bisector A

$-x - y + 5 = 0$

$7x + y + 14 = 0$

$(-1)(7) + (-1)(1) < 0$

$\Rightarrow$ acute angle bisector of A with (+) sign.

$$\frac{-x-y+5}{\sqrt{2}} = \frac{7x+y+15}{5\sqrt{2}} \Rightarrow 12x + 6y = 11$$

Bisector B

$-x - y + 5 = 0$ & $-x - 7y + 7 = 0$

$(-1)(-1)(-7) > 0$

$\Rightarrow$ acute angle bisector of B with (–) sign

$$\frac{-x-y+5}{\sqrt{2}} = -\frac{(-x-7y+7)}{5\sqrt{2}}$$

$\Rightarrow 6x + 12y = 32 \Rightarrow 3x + 6y = 16$

Bisector-C

$-x - 7y + 7 = 0$

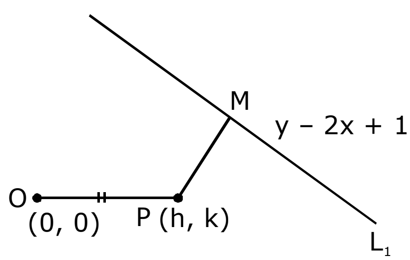
$7x + y + 14 = 0$

$(-1)(7) + (-7)(1) < 0$

External bisector of C is acute with (+) sign

$$\frac{-x-7y+7}{5\sqrt{2}} = +\frac{7x-y+14}{5\sqrt{2}} \Rightarrow 8x+8y+7=0$$

Sol.7 $(PM)^2 = (OP)^2$



$$\left(\frac{k-2h+1}{\sqrt{5}}\right)^2 = \left(\sqrt{(h-0)^2+(k-0)^2}\right)$$

$\Rightarrow (k - 2h + 1)^2 = 5(h^2 + k^2)$

$\Rightarrow (y - 2x + 1)^2 = 5(x^2 + y^2)$

Locus

$\Rightarrow x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$

Locus C at by y= 2x at Q & R

$\Rightarrow (1)^2 = 5(x^2 + 4x^2)$

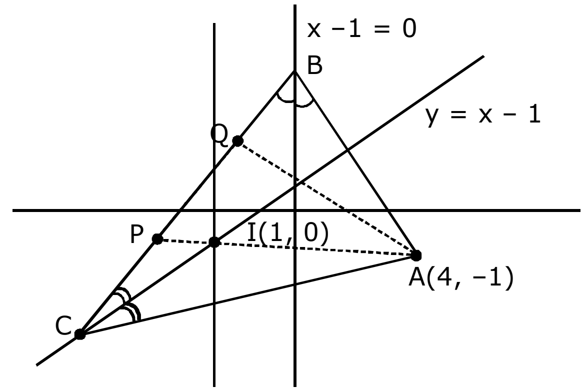
$$\Rightarrow x = \pm\frac{1}{5} \Rightarrow y = \pm\frac{2}{5}$$

$$Q\left(\frac{1}{5}, \frac{2}{5}\right) \text{ \& } R\left(-\frac{1}{5}, \frac{-2}{5}\right)$$

$$\because QR = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{2}{\sqrt{5}}$$

Mid point of QR is (0, 0)

Sol.8 Image of A w.r.t to $x - 1 = 0$ is P



$$\frac{x-4}{1} = \frac{y+1}{0} = \frac{-2(4-1)}{1^2+O^2} = -6$$

P(–2, –1)

Image of A w.r. to $x - y - 1 = 0$ is Q

$$\frac{x-y}{1} = \frac{y+1}{-1} = -\frac{-2(4+1-1)}{1^2+(-1)^2} = -4$$

$x = 0, y = 3$ Q (0, 3)

Line BC is same PQ

$$y - 3 = \frac{4}{2}x \Rightarrow 2x - y + 3 = 0$$

$\therefore$ B (1, 5) & C (–4, –5)

$$\text{Line AB is } y + 1 = \frac{6}{-3}(x - 4)$$

$\Rightarrow 2x + y - 7 = 0$

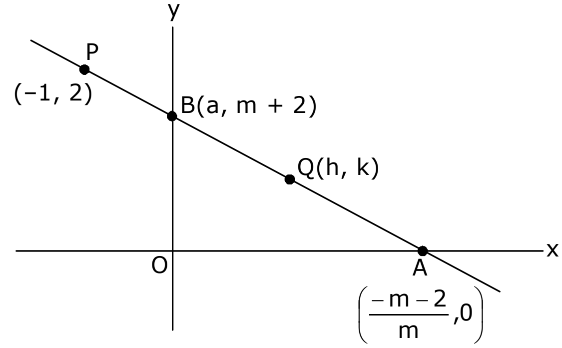
$$\text{Line AC is } y + 1 = \frac{-4}{-8}(x - 4)$$

$\Rightarrow x - 2y = 6 = 0$

Sol.9 Line

$y - 2 = m(x + 1)$

$$B(0, m + 2), A\left(\frac{-m+2}{m}, 0\right)$$



Let

$$\frac{PA}{PB} = \frac{\lambda}{1}$$

P divides AB externally in ratio $\lambda : 1$

$$2 = \frac{\lambda(m+2)-0)}{\lambda-1} \Rightarrow \lambda = \frac{-2}{m}$$

Q divides AB internally in ratio $\lambda : 1$

$$h = \frac{-m-2}{m\frac{(m-2)}{m}} \ \& \ k = \frac{-2(m+2)}{m\frac{m-2}{m}}$$

$$h = -\frac{(m+2)}{m-2} \ \& \ k = -2\frac{(m+2)}{m-2}$$

$k = -2\,(-h)$

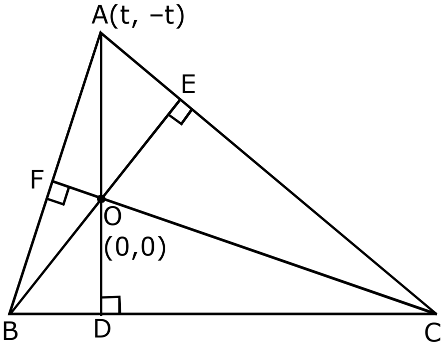
$k = 2h$

Locus $y = 2x$

Sol.10 AD : $x + y = 0$

BE : $x - 4y = 0$

CF : $2x - y = 0$



Line AC is $\perp$ BE passing A

$4x + y - 3t = 0$

Line AB is $\perp$ CF passing A

$x + 2y + 1 = 0$

B & C are intersection point of AB & BE and AC × CF respectively

$$B\left(-\frac{2t}{3}, \frac{-t}{6}\right) \ \& \ C\left(\frac{t}{3}, t\right)$$

Let centroid is (h, k)

$$\therefore \ 3h = t - \frac{2t}{3} + \frac{t}{3} \ \& \ 3k = -t - \frac{t}{6} + t$$

$$3h = \frac{5t}{6} \ \& \ 3k = -\frac{t}{6}$$

$3h = 5\,(-3k) \Rightarrow h = -5k \Rightarrow x + 5y = 0$

Sol.11 Let line $y = mx$ have $\{y = m_1x \ \& \ y = m_2x\}$ distance from (x_1, y_1) is δ

$$\delta = \frac{|mx_1 - y_1|}{\sqrt{m^2+1}} \Rightarrow \delta^2(m^2+1) = (mx_1 - y_1)^2$$

$\Rightarrow \delta^2m^2 + \delta^2 = m^2x_1^2 + y_1^2 - 2m_1x_1y_1$

$\Rightarrow m^2(x_1^2 - \delta^2) - 2mx_1y_1 + y_1^2$

$- d^2 = 0 \begin{cases} m_1 \\ m_2 \end{cases}$

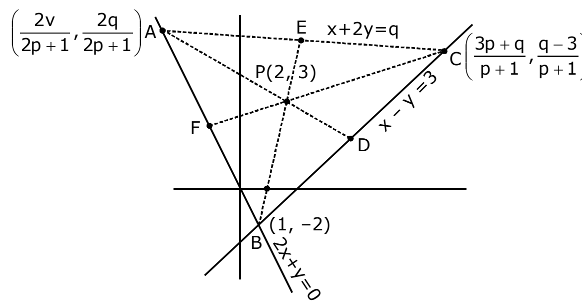
$$m_1 + m_2 = \frac{mx_1y_1}{x_1^2 - \delta^2} \quad \{m_1 + m_2 = \frac{-2h}{b}$$

$$m_1m_2 = \frac{y_1^2 - \delta^2}{x_1^2 - \delta^2} \ \{m_1m_2 = \frac{a}{b}$$

$$\left(\frac{y}{x}\right)^2 - \frac{2x_1y_1}{x_1^2 - \delta^2}\left(\frac{y}{x}\right) + \frac{(y_1^2 - \delta^2)}{x_1^2 - \delta^2} = 0$$

$x^2(y_1^2 - \delta^2) - 2x_1y_1xy + (x_1^2 - \delta^2)\,y^2 = 0$

Sol.12



(a) $$d\frac{-q}{2p-1} + \frac{3p+q}{p+1} + 1 = 3.2$$

$$\Rightarrow -\left(\frac{q}{2p+1}\right) + \frac{3(p+q)+(q-3)}{(p+1)} = 5$$

$$\Rightarrow -\left(\frac{q}{2p-1}\right) + \left(\frac{q-3}{p+1}\right) = 2 \quad ....(i)$$

$$\& \left(\frac{2q}{2p-1}\right) + \left(\frac{q-3}{p+1}\right) - 2 = 3.3$$

$$\Rightarrow 2\left(\frac{q}{2p-1}\right) + \left(\frac{q-3}{p+1}\right) = 11 \ ....(ii)$$

Solve (i) & (ii) $\frac{q}{2p-1} = 3 \ \& \ \frac{q-3}{p+1} = 5$

$\Rightarrow q = 5p + 8 = 6p - 3$

$\Rightarrow p = 11 \ \& \ q = 63 \Rightarrow p + q = 74$

(b) P(2, 3) orthocentre

$m_{BP}\, m_{AC} = -1$

$\frac{5}{1} \cdot \left(-\frac{1}{p}\right) = -1 \qquad \Rightarrow p = 5$

$PD \perp AC$

$\therefore$ equation of PD $x + y = \lambda$

Solve the $2x + y = 0$

a $(-5, 10)$ satisfy BC

$-5 + 10p = q$

$\Rightarrow q = 45 \qquad \therefore p + q = 50$

(c) Line PD

$\Rightarrow x + y = 5$ & D $\left(\frac{4p+q+1}{2(P+1)}, \frac{q-2p-5}{2(P+1)}\right)$

$\because$ PD $\perp$ BC dis mid point of BC satisfy line PF

$\therefore 2P - q - 1 - 4q + 8p - 4 + 16P - 8 = 0$

$\Rightarrow 26p - 5q = 13 \qquad$ ....(ii)

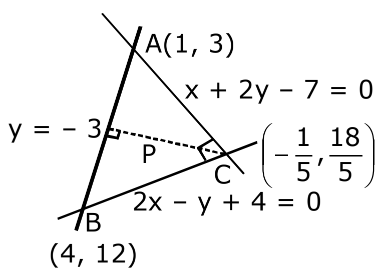
Solve (i) & (ii) & get P = 8 q = 39

p + q = 47

Sol.13 $2x^2 + 3xy - 2y^2 - 10x + 15y - 28 = 0$

& $y = 3x$

a + b $\Rightarrow$ $\perp$ lines



$(x + 2y)(2x - y) + 15y - 25 = 0$

$(x + 2y - 7)(2x - y + 4) = 0$

$x + 2y - 7 = 0$

$\underline{4x - 2y + 8 = 0}$

$5x + 1 = 0 \qquad \Rightarrow x = \frac{1}{5} \;\; y = \frac{18}{5}$

& $x + 6x - 7 = 0 \quad \Rightarrow x = 1, y = 3$

& $2x - 3x + 4 = 0 \;\; x = 4, y = 12$

(a) Slopes 3, 2, $-\frac{1}{2}$

m_1, m_2, m_3

$\tan B = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{3-2}{1+6} = \frac{1}{7}$

$\Rightarrow \cot B = 7$

tan C = N.D. $\quad \Rightarrow \cot C = 0$

$\cot A + \cot B + \cot C = +\frac{1}{7} + 7 + 0 = \frac{50}{7}$

(b) $\Delta = \frac{1}{2}(AB) \times p$

$\therefore AB = \sqrt{9+81} = 3\sqrt{10}$

$p = \left|\frac{\left(\frac{-3}{5} - \frac{18}{5}\right)}{\sqrt{3^2 + 1^2}}\right| = \frac{21}{5\sqrt{10}}$

$\Delta = \frac{1}{2} \times 3\sqrt{10} \times \frac{21}{5\sqrt{10}} = \frac{63}{10}$

(c) $r = \frac{\Delta}{s} \quad AB = 3\sqrt{10}$

$AC = \sqrt{\left(\frac{6}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{36+9}{(5)^2}}$

$= \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$

$BC = \sqrt{\left(\frac{21}{5}\right)^2 + \left(\frac{42}{5}\right)^2}$

$2.S = 3\sqrt{10} + \frac{3\sqrt{5}}{5} + \frac{21\sqrt{5}}{5}$

$= \frac{15\sqrt{10} + 24\sqrt{5}}{5} \qquad \Rightarrow S = \frac{3(5\sqrt{10} + 8\sqrt{5})}{10}$

$r = \frac{63}{10} \times \frac{10}{3(5\sqrt{10} + 8\sqrt{5})}$

$= \frac{21}{(8\sqrt{5} + 5\sqrt{10})} \times \frac{(5\sqrt{10} + 5\sqrt{10})}{(8\sqrt{5} - 5\sqrt{10})}$

$= \frac{21(8\sqrt{5} - 5\sqrt{10})}{64.5 - 25.10} = \frac{21(8\sqrt{5} - 5\sqrt{10})}{5(64-50)}$

$= \frac{321}{5 \times 14}(8\sqrt{5} - 5\sqrt{10}) = \frac{3}{10}(8\sqrt{5} - 5\sqrt{10})$

Sol.14 Let chorcd is y = mx + c

$\Rightarrow \dfrac{y - mx}{c} = 1$

honoginizatoin with

$3x^2 - y^2 - 2x + 4y = 0$

$3x^2 - y^2 - 2x\left(\dfrac{y - mx}{C}\right) + 4y\left(\dfrac{y - mx}{c}\right) = 0$

$\Rightarrow x^2\left(3 + \dfrac{2m}{c}\right) + \left(\dfrac{-2}{c} - \dfrac{4m}{c}\right)xy + y^2\left(-1 + \dfrac{4}{c}\right) = 0$

represent $\perp$ lines

$\therefore\ 3 + \dfrac{2m}{c} - 1 + \dfrac{4}{c} = 0$

$\Rightarrow 1 + \dfrac{m+2}{c} = 0 \Rightarrow c = -m - 2$

put in equation of chord

$\Rightarrow (y + 2) = m(x - 1)$

passes through always (1, –2)

Homogenization with

$3x^2 + 3y^2 - 2x + 4y = 0$

$3x^2 + 3y^2 - 2x\left(\dfrac{y - mx}{c}\right) + 4y\left(\dfrac{y - mx}{c}\right) = 0$

$\Rightarrow x^2\left(3 + 2\dfrac{m}{c}\right) + \left(\dfrac{-2 - 4m}{c}\right)xy + y^2\left(3 + \dfrac{y}{c}\right) = 0$

Represent $\perp$ lines

$\therefore\ 3 + \dfrac{2m}{c} + 3 + \dfrac{4}{c} = 0$

$6c + 2m + 4 = 0$

$3c + m + 2 = 0$

$c = -\dfrac{m}{3} - \dfrac{2}{3}$

Put in eqaution of chord

$y = mx - \dfrac{m}{3} - \dfrac{2}{3}$

$\left(y + \dfrac{2}{3}\right) = m\left(x - \dfrac{1}{3}\right)$

always passes through

$\left(\dfrac{1}{3}, -\dfrac{2}{3}\right)$

Yes given result hold with

$3x^2 + 3y^2 - 2x + 4y = 0$

Sol.15 Line passes through (1, 0)

$y = m(x - 1) \Rightarrow y = mx - x \Rightarrow \dfrac{mx - y}{m} = 1$

Homogination with $x^2 + y_2 + 6x - 10y + 1 = 0$

$x^2 + y^2 + 6x\left(\dfrac{mx - y}{m}\right) - 10y\left(\dfrac{mx - y}{m}\right)$

$+ \left(\dfrac{mx - y}{m}\right)^2 = 0 \qquad ....(i)$

$\Rightarrow$ Coeff. of $x^2 = 1 + 6 + 1 = 8$

& coeff of $y^2 = 1 + \dfrac{10}{m} + \dfrac{1}{m^2}$

If equatoin (i) subtends right angle of origin

$8 + 1 + \dfrac{10}{m} + \dfrac{1}{m^2} = 0$

$\left(\dfrac{1}{m^2}\right) + 10\left(\dfrac{1}{m}\right) + 9 = 0$

$\left(\dfrac{1}{m} + 9\right)\left(\dfrac{1}{m} + 1\right) = 0$

$m = -\dfrac{1}{9}, m = -1$

lines are

$y = -\dfrac{1}{9}(x - 1)$ or $y = -1(x - 1)$

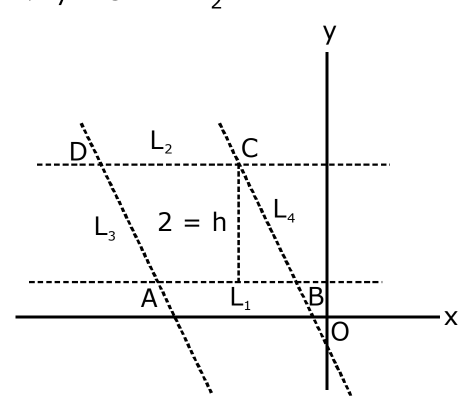
$x + 9y - 1 = 0$ or $x + y - 1 = 0$

Sol.16 $y^2 - 4y + 3 = 0$

$(y - 3)(y - 1) = 0$

$\Rightarrow y = 1 \qquad L_1$

$\Rightarrow y = 3 \qquad L_2$



$x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$

$(x + 2y + A)(x + 2y + B) = 0$

$\Rightarrow x + 2y + 4 = 0 \quad L_3$

$x + 2y + 1 = 0 \quad L_4$

A(–6, 1), B (–3, 1)

C(–7, 3), D(–10, 3)

AB = 3, hight h = 2

Area of parallelogram = base × hight

= 3 × 2 = 6 sq. units

Diagonals

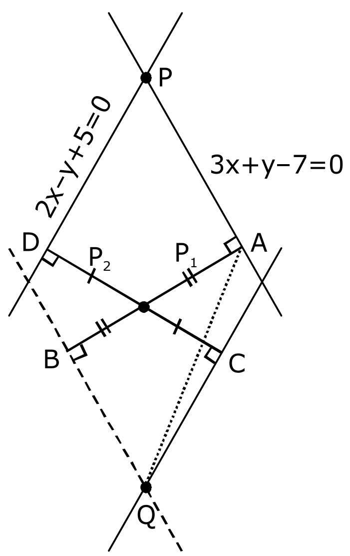
$AC = \sqrt{1^2 + q^2} = \sqrt{5}$

$BD = \sqrt{7^2 + 2^2} = \sqrt{53}$

Sol.17 $6x^2 - 16y - y^2 + x + 12y - 35 = 0$

$(2x - y + A)(3x + y + B) = 0$

$\Rightarrow (2x - y + 5)(3x + y - 7) = 0$



$p_1 = \dfrac{5}{\sqrt{5}} = \sqrt{3}$

$p_2 = \dfrac{|-7|}{\sqrt{10}} = \dfrac{7}{\sqrt{10}}$

Line CD

$\dfrac{x - 0}{\frac{2}{\sqrt{5}}} = \dfrac{y - 0}{\frac{1}{\sqrt{5}}} = \pm \dfrac{7}{\sqrt{10}}$

Point A or B $\left(\pm\dfrac{21}{10}, \pm\dfrac{7}{10}\right)$

A is $\left(\dfrac{21}{10}, \dfrac{7}{10}\right)$ & ϕ is $\left(-\dfrac{21}{10}, -\dfrac{7}{10}\right)$

Equation of side is || to $2x + y + \lambda = 0$

passes through C

$\Rightarrow \lambda = -5$

$2x - y - 5 = 0$

equation of side is || to $3x + y - 7 = 0$ &

passes through B $\left(\dfrac{-21}{10}, \dfrac{-7}{10}\right)$

$3x + y + \mu = 0 \quad \Rightarrow \mu = 47$

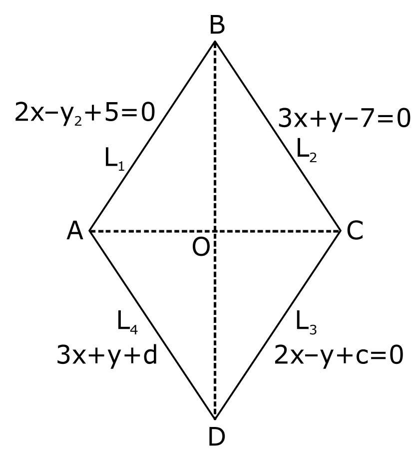
$3x + y + 7 = 0$

combined equation is

$(2x - y - 5)(3x + y + 7) = 0$

$\Rightarrow 6y^2 - xy - y^2 - x - 12y - 35 = 0$

Aliter :



diagonal AC

$L_1L_2 - L_3L_4 = 0$

Passing through (0, 0)

$-35 - cd = 0 \Rightarrow cd = -35$ & diagonal BD

$L_1L_4 - L_2L_3 = 0$

passing though (0, 0)

$5d - (-7c) = 0 \quad \Rightarrow 5d + 7c = 0$

$5\left(\dfrac{-35}{c}\right) + 7c = 0$

$\Rightarrow c^2 = 25 \Rightarrow 25 \Rightarrow c = \pm 5$ & $d = \pm 7$

for required line c = – 5, d = + 7

$(2x - y - 5)(3x + y + 7) = 0$

$\Rightarrow 6x^2 - xy - y^2 - x - 12y - 35 = 0$